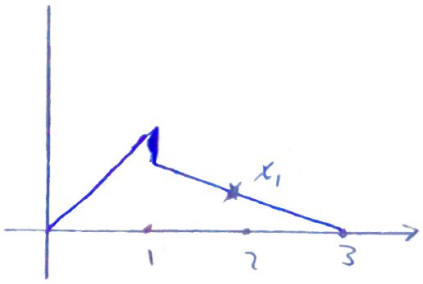
7/11/2015 السبت د. سيد سلام محاضرة 6

for the previous example (Lec 5)

$0 \le x \le 1$

$$F(x) = \int_0^x x\,dx = \frac{x^2}{2}\Big|_0^x = \frac{x^2}{2}$$

$$f(x)\Big|_{x=1} = \int_0^1 x\,dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$$



$1 \le x \le 3$

$$F(x) = \begin{cases} \sum [p(x)] \text{ OR} \\ \int [p(x)\,\Pi(x)] \end{cases}$$

$$\Rightarrow F(x) = \int_{-\infty}^{x} p(x)\,dx = \int_{-\infty}^{0} p(x)\,d(x) + \int_0^1 p(x_1)\,dx + \int_1^x p(x_2)\,dx$$

$$= 0 + \int_0^1 x\,dx + \int_1^x \left(\frac{3-x}{4}\right) dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \left[\frac{1}{4}\left(3x - \frac{x^2}{2}\right)\right]_1^x$$

$$= \frac{1}{2} + \frac{1}{4}\left[3x - \frac{x^2}{2}\right] - \left[\frac{1}{4}\left(3 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + \frac{1}{4}\left\{3x - \frac{x^2}{2} - \frac{1}{4}\left(\frac{5}{2}\right)\right\}$$

$$= \frac{1}{4}\left[3x - \frac{x^2}{2} - \frac{5}{8}\right] + \frac{1}{2}$$

$$= \frac{1}{4}\left[3x - \frac{x^2}{2} - \frac{5}{8} + \frac{16}{8}\right]$$

$$= \frac{1}{4}\left[3x - \frac{x^2}{2} + \frac{11}{8}\right]$$

$$F(x) = \begin{cases} 0 & -\infty \le x \le 0 \\ x^2/2 & 0 \le x \le 1 \\ -1/8 + 1/4(3x - x^2/2) & 1 \le x \le 3 \\ 1 & 3 \le x \end{cases}$$

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* Mathematical Expectation

If X is a random variable with probability fn, then the mean or expectation (or expected value) of X, denoted by $E(x)$ or μ_x or simply E or μ, is defined by

$$E(x) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n (P x_n)$$

$$E(x) = \sum_{i=1}^{n} x_i P(x_i) \qquad x \text{ is discrete}$$

$$E(x) = \int_{-\infty}^{\infty} x P(x)\, dx \qquad \text{continuous}$$

Generally, if $\psi(x)$ is a function of x where x is a random variable, then:

$$E[\psi(x)] = \int_{-\infty}^{\infty} \psi(x) P(x)\, dx \qquad \text{continuous}$$

$$E[\psi(x)] = \sum_{-\infty}^{\infty} \psi(x) P(x) \qquad \text{discrete}$$

E is usually called the first moment about the origin. that is, $E \Rightarrow$

That is, E is the weighted average of the possible values of x, each value weighted by its probability

Example:- Let x be a random variable taking the values $x_1, x_2, \dots x_n$ with probabilities $\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \dots \frac{1}{n}$. Find E(x)

$$E(x) = x_1 \cdot \frac{1}{n} + x_2 \frac{1}{n} + \dots + x_n \frac{1}{n} = \left[\frac{x_1 + x_2 + \dots + x_n}{n}\right]$$

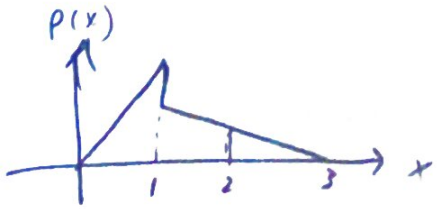
= the mean

First
mean = moment = expectation

Example:-

Find E(x) if P(x) is defined as

$$P(x) \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ \frac{3-x}{4} & 1 \le x \le 3 \\ 0 & 3 \le x \end{cases}$$



Solution :-

E(x) = mean = μ_x = Expectation = first moment

$$= \int_{-\infty}^{\infty} x\, P(x)\, dx = \int_{-\infty}^{0} x\, P(x)\, dx + \int_{0}^{1} x\, P(x)\, dx$$

$$+ \int_{1}^{3} x\, P(x)\, dx + \int_{3}^{\infty} x\, P(x)\, dx$$

$$= \int_{0}^{1} x \cdot x\, dx + \int_{1}^{3} x \cdot \frac{3-x}{4}\, dx = \frac{7}{6}$$

$$E(x^2) = \int_0^1 x^2 \cdot x\,dx + \int_1^3 x^2 \left(\frac{3-x}{4}\right) dx$$

$$= 1.75$$

Theorem If c is a constant with respect to x, then $\boxed{E(c) = c}$

$$E(c) = \int_{-\infty}^{\infty} c \cdot p(x)\,dx = c \cdot 1 = c$$

Also $E[c\,\psi(x)] = c\,E[\psi(x)]$

\# Example: $E[ax + b] = a\,E(x) + b$

$$E[3 - 2x] = 3 - 2E(x)$$

The second moment about the origin $= E(x^2)$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 p(x)\,dx \quad , \quad \sum_{-\infty}^{\infty} x^2 \cdot p(x)$$

The rth moment about the origin $= E(x^n)$

The 1st moment about the mean $= E(x-\mu)$

$=$ 2nd " " " " $= \mu_2 = E(x-\mu)^2$

the rth " " " " $= \mu_r = E(x-\mu)^r$

⟹ Turn over

mean = $E(x) = E = \mu_x = \mu = \bar{m}$ = Expectation

* Variance and Standard deviation

Var, S.D
↓ ↓
σ^2, σ

The second moment about μ is given by

$$\mu_2 = E(x-\mu)^2$$

$$= E(x^2 - 2\mu x + \mu^2)$$

$$= E(x^2) - 2\mu E(x) + \mu^2$$

$$= E(x^2) - 2\mu\mu + \mu^2$$

$$= E(x)^2 - \mu^2 = \text{seond moment about mean}$$

$$= \text{Second} \lesseqgtr \text{origin} - (\text{mean})^2$$

μ_2 is called the variance of X (Var. X).

It measures the dispersion of X. The Standard deviation of X, denoted by σ_x, is the non-negative square root of (var x)

$$\text{Var. } X = \sigma^2 X \quad , \quad \sigma_x = +\sqrt{\text{Var } x}$$

* Prove

1) $\text{Var}(c) = 0$

$\text{Var}(aX) = a^2 \text{ Var}(x)$

$\text{Var}(aX + b = a^2 \text{ Var}(x)$